UNIVERSITY OF TWENTE.

EXAM: Causal Inference

July 1st, 2025, 13.45-16.45 h.

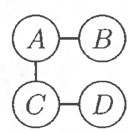
Instructor: Wouter Koolen

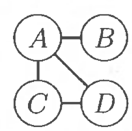
Please answer the questions on separate sheets of paper. Do not forget to number the sheets and to write your name on each of them.

Exercises

1. [2pt] Suppose consumption of vitamins has a positive causal effect on life expectancy. Yet in an observational study we see a negative correlation. How can that be explained?

2. Consider the following two undirected graphical models.

(a) 

(b) 

   (a) [1pt] Does a fully independent joint distribution of A, B, C, D satisfy the constraints expressed by the undirected graphical model (a)?
   (b) [1pt] Does any P that satisfies (b) satisfy (a) as well? What about the converse?
   (c) [1pt] Give a conditional independence statement that is true under (b).
   (d) [1pt] Give a conditional independence statement that is true under (a) but does not follow from (b).

3. [3pt] Consider observed binary variables L, V, A, Y and unobserved potential outcomes Y^0, Y^1 satisfying the standard assumptions (P), (C), (CE) recalled on the cheat sheet. We want to estimate the conditional causal effect CATE $= \mathbb{E}[Y^1 - Y^0 | V = v]$. In the lectures we assumed that the group label V is a coarsening of the covariate L. Here instead we assume that $Y^a \perp\!\!\!\perp V | L$ for both treatments $a \in \{0, 1\}$. Show that CATE is identifiable.

4. Consider four binary variables: treatment A, outcome Y and potential outcomes Y^0 and Y^1 (we ignore covariates in this question). We assume that treatment A and outcome Y are observed, while the potential outcomes Y^0 and Y^1 are not. Let us impose the following constraints on the joint distribution $P(A, Y, Y^0, Y^1)$:

(C) Consistency: $Y = Y^A$

(CE) Conditional exchangeability: $Y^a \perp\!\!\!\perp A$ for each $a \in \{0, 1\}$

(SI) Simple independence: $Y^0 \perp\!\!\!\perp Y^1$

(a) [1pt] Do these constraints imply $Y \perp\!\!\!\perp A$? Prove or give a counterexample.

(b) [1pt] Show that the joint distribution $P(A, Y, Y^0, Y^1)$ is uniquely determined by its three marginal probabilities $\alpha = P(A = 1)$ and $\gamma^a = P(Y^a = 1)$ for $a \in \{0, 1\}$.

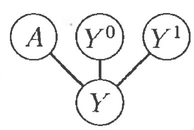
(c) [1pt] With the same marginal notation as above, show that the marginal distribution $P(A, Y)$ of the observed variables A and Y is

	$Y = 1$	$Y = 0$
$A = 1$	$\alpha\gamma^1$	$\alpha(1 - \gamma^1)$
$A = 0$	$(1 - \alpha)\gamma^0$	$(1 - \alpha)(1 - \gamma^0)$

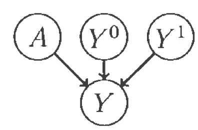
(d) [1pt] Can any candidate joint distribution $Q(A, Y)$ of observed variables A, Y arise from a distribution P of the above form?

(e) [2pt] Consider the two graphical models below. For each, determine if it is a faithful representation of the constraints on P.

(a)

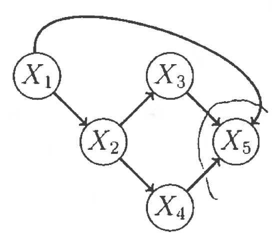


(b)



5. Consider the Linear Structural Equation Model (left) and corresponding DAG (right)

$$
\begin{aligned}
X_1 &= \epsilon_1 \\
X_2 &= \beta_{21} X_1 + \epsilon_2 \\
X_3 &= \beta_{32} X_2 + \epsilon_3 \\
X_4 &= \beta_{42} X_2 + \epsilon_4 \\
X_5 &= \beta_{51} X_1 + \beta_{53} X_3 + \beta_{54} X_4 + \epsilon_5
\end{aligned}
$$



where $\epsilon_2, \ldots, \epsilon_5$ are i.i.d. standard normal.

(a) [2pt] Give the Markov equivalence class and the corresponding CPDAG.

(b) [1pt] Compute the causal effect per unit intervention $\frac{\partial}{\partial x} \mathbb{E}[X_5 | do(X_1 = x)]$.

(c) [1pt] Is $\{X_1, X_3\}$ an adjustment set (aka set of control variables) for the effect of X_4 on X_5?

(d) [1pt] Assume that $\beta_{21} \neq 0$. Show that X_1 is an instrumental variable for the effect of X_2 on X_4. Do so by proving that $\text{Cov}(X_4 - \beta_{42} X_2, X_1) = 0$ yet $\text{Cov}(X_2, X_1) \neq 0$.