Reexam Continuous Optimization

16 February 2026, 10.00–13.00

This closed-book exam consists of 5 questions. Please start each question on a new A4 sheet, write legibly, and hand in your work with the solutions in the correct order. In total, you can obtain 90 points. The final grade is $1 + \#\text{points}/10$ rounded to half integers (except 5.5). Good luck!

1. Consider the function f defined by

$$f(x_1, x_2) = x_1^2 + \alpha x_2^2 + \cos(x_2),$$

where $\alpha \in \mathbb{R}$ is a parameter.

   (a) (5 points) Suppose we apply gradient descent with exact line search starting from the point $(1, 0)$. Compute the next iterate.

   (b) (5 points) For which values of α is the function f convex? Explain your answer.

   (c) (5 points) Suppose α is very large, say, $\alpha = 10^4$. Discuss the convergence behavior and convergence speed of gradient descent with exact line search starting from an arbitrary point near the origin.

2. Let $v \in \mathbb{R}^n$, $a \in \mathbb{R}^n$ a unit vector, and $b \in \mathbb{R}$. Consider the problem

$$\begin{array}{ll} \text{minimize} & \|x - v\|_2^2 \\ \text{subject to} & a^\mathsf{T} x \leq b, \end{array}$$

   (a) (10 points) Derive the Lagrangian, Lagrange dual function, and Lagrange dual problem.

   (b) (5 points) Show strong duality holds without computing the primal and dual optimal solutions.

   (c) (5 points) Find the optimal dual solution.

3. (a) (7 points) Suppose we apply Fibonacci search with $k = 6$ to a unimodal function with initial bracket $[0, 10]$ (this requires 6 function evaluations). What is the size of the final bracket?

   (b) (8 points) Give the definition of convergence order/rate and give an example of a sequence that has cubic convergence (convergence of order 3).

4. (a) (15 points) Consider the function

$$F(x_1, x_2) = x_1 e^{x_1 x_2} + x_2$$

where we view addition, multiplication, and the exponential as elementary functions. Compute $\nabla F(4, 0)$ using reverse-mode automatic differentiation by drawing the appropriate diagrams.

   (b) (5 points) Suppose we have a function $f \colon \mathbb{R}^{100} \to \mathbb{R}^2$ that is composed of elementary functions. How costly is computing the Jacobian of f using reverse-mode automatic differentiation compared to evaluating this function?

5. (a) (15 points) Consider the linear program

$$\begin{array}{ll} \text{minimize} & c^\mathsf{T} x \\ \text{subject to} & Ax = b \\ & x \geq 0. \end{array}$$

Give the centralizer problem (sometimes called the barrier problem) and compute the Hessian of the objective function of the centralizer problem. Explain why using a single centralizer problem and Newton's method with equality constraints is not a viable method for solving this problem.

(b) (5 points) For what types of problems are interior-point methods preferable to ADMM, and conversely, for what types of problems is ADMM preferable to interior-point methods?

End of test