Kenmerk: EWI2016/dmmp/013/BM

Exam Limits to Computing (201300042)

Thursday, November 3, 2016, 8:45 – 11:45

- Use of calculators, mobile phones, etc. is not allowed!
- This exam consists of five problems.
- Please start a new page for each problem.
- The total number of points is 63+7 = 70. The distribution of points is according to the following table.

1a: 6	2: 15	3a: 4	4a: 4	5a: 3
1b: 7		3b: 5	4b: 4	5b: 3
1c: 3				5c: 3
				5d: 3
				5e: 3

1. Decidability and Recursive Enumerability

Consider the following problem:

$$\text{INJECTIVE} = \{g \in \mathcal{G} \mid \varphi_g \text{ is injective}\}.$$

Recall that injective for partial functions means the following: A partial function $f : \mathbb{N} \to \mathbb{N}$ is injective if, for all $x, y \in \text{dom}(f)$, we have that $f(x) = f(y)$ implies $x = y$.

(a) *(6 points)* Is INJECTIVE $\in$ REC? Prove your answer.

(b) *(7 points)* Prove that INJECTIVE $\in$ co-RE.

(c) *(3 points)* Is INJECTIVE $\in$ RE? Prove your answer.

2. NP-Completeness

A 3-uniform hypergraph $H = (V, E)$ consists of a set V of vertices and a set E of edges. The difference to a graph is that every edge connects exactly three vertices. This means that if $e \in E$, then $e = \{u, v, w\}$ for distinct $u, v, w \in V$.

A 3-coloring of a 3-uniform hypergraph $H = (V, E)$ is a function $\mathrm{col} : V \to \{1, 2, 3\}$ such that for all edges $e \in E$, all three nodes of E are colored differently. In other words, if $e = \{u, v, w\}$, then $\{\mathrm{col}(u), \mathrm{col}(v), \mathrm{col}(w)\} = \{1, 2, 3\}$.

For instance, if $V = \{a, b, c, d\}$ and $E = \big\{\{a, b, c\}, \{b, c, d\}\big\}$, then $\mathrm{col}(a) = \mathrm{col}(d) = 1$ and $\mathrm{col}(b) = 2$ and $\mathrm{col}(c) = 3$ is a 3-coloring of H.

Let

$$\textsc{Hyper3Coloring} = \big\{H \mid H \text{ is a 3-uniform hypergraph and 3-colorable}\big\}.$$

Prove that $\textsc{Hyper3Coloring}$ is NP-complete.

Hint: 3-$\textsc{Coloring} = \{G \mid G \text{ is a graph and 3-colorable}\}$ is NP-complete.

3. Complexity Classes 1

Let $A, B \subseteq \Sigma^\star$ be two decision problems. A linear-time many-one reduction is a function $f : \Sigma^\star \to \Sigma^\star$ with the following properties:

(i) For all $x \in \Sigma^\star$, the function value $f(x)$ can be computed in linear time, i.e., in time $O(|x|)$.

(ii) For all $x \in \Sigma^\star$, we have $x \in A$ if and only if $f(x) \in B$.

We write $A \leq_{\text{lin}} B$ if there is a linear-time many-one reduction from A to B.

(a) *(4 points)* Prove the following: For

- all time-constructible functions t with $t(n) \geq n$ for all n and
- all decision problems A and B

the following holds: If $A \leq_{\text{lin}} B$ and $B \in \mathsf{DTime}(O(t))$, then $A \in \mathsf{DTime}(O(t))$.

(b) *(5 points)* Prove or disprove: there exists a problem $B \in \mathsf{P}$ such that $A \leq_{\text{lin}} B$ for all $A \in \mathsf{P}$. In other words, there exists a problem B that is P-complete with respect to $\leq_{\text{lin}}$.

4. Complexity Classes 2

For this exercise, let $\mathsf{UnionP} = \mathsf{NP} \cup \text{co-}\mathsf{NP}$.

(a) *(4 points)* Is UnionP closed under polynomial-time many-one reductions, i.e., is it true that $A \leq_{\mathrm{P}} B$ and $B \in \mathsf{UnionP}$ implies $A \in \mathsf{UnionP}$?

(b) *(4 points)* Prove the following: If there exists a problem Q that is UnionP-complete with respect to polynomial-time many-one reductions, then $\mathsf{NP} = \text{co-}\mathsf{NP}$.

5. Questions

Are the following statements true or false? Justify your answers.

(a) *(3 points)* If $\mathsf{NP} = \mathsf{NL}$, then $\mathsf{P} = \mathsf{PSPACE}$.

(b) *(3 points)* All decision problems in NP are decidable.

(c) *(3 points)* UCONN $\in \mathsf{NL}$.

(d) *(3 points)* For all sets $A, B \subseteq \mathbb{N}$, the following holds: If $A \cup B \in \mathsf{REC}$, then $A, B \in \mathsf{REC}$.

(e) *(3 points)* For all sets $A, B \subseteq \mathbb{N}$, the following holds: If $A \in \mathsf{REC}$ and $B \in \mathsf{RE}$, then $A \setminus B \in \text{co-}\mathsf{RE}$.