Resit ADM, Module 7, Codes 202500366 & 202001360
Algorithmic Discrete Mathematics
June 15th, 2026, 8:45–10:45

Answers to questions 1-5 need to be motivated, arguments and proofs must be complete. A cheat-sheet is provided with the exam. Other aids (including the use of electronic devices) are <u>not</u> allowed.

For information: The total number of points is 36. Your grade is determined as follows:

- Students registered for course code 202500366 (standalone 3EC course): $1 + 0.25x$, with x being the number of obtained points. That means you need 18 points to get a 5.5.
- **Repeat students** registered for course code 202001360 (5EC course, combined with ADS):

$$\frac{X_{\text{ADS}}}{20} + \frac{5X_{\text{ADM}}}{36} + 1,$$

  where X_{ADS} and X_{ADM} are the number of points obtained in the ADS and ADM exams respectively. Note that for repeat students (registered for Course Code 202001360) Question 6(e) (indicated with a "$(*)$") will not be graded.

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1. (2+3 points)
   (a) Consider $a, b, c \in \mathbb{Z}$ with $a \mid bc$. Prove or disprove that: $a \mid b$ or $a \mid c$.
   (b) Prove Euclid's Lemma: Consider $a, b \in \mathbb{Z}$ and p prime with $p \mid ab$. Then $p \mid a$ or $p \mid b$ (or both).
2. (3+2 points)
   (a) Consider a simple graph $G_1 = (V_1, E_1)$ with average degree greater than 2. Prove that G_1 contains a cycle.
   (b) Consider a simple connected graph $G_2 = (V_2, E_2)$ with $|V_2| = |E_2|$. Prove that G_2 contains *exactly one* cycle.
3. (6 points) Consider a simple, capacitated network $G = (V, E, c)$, where V is the set of vertices, $s, t \in V$, E is the set of directed edges, and $c(e) \in \mathbb{Z}$ for all $e \in E$ are the integer edge capacities. Let $|V| = n$ and $|E| = m$. Suppose you are given a maximum (s,t)-flow f in G, and that f was computed by the Ford-Fulkerson algorithm.

   Consider now a simple, capacitated network $G' = (V, E, c')$ with $c'(e) = c(e), \forall e \in E \setminus \{e^*\}$ and $c'(e^*) = c(e) + 5$. In other words G' is identical to G, except that the capacity of a specific edge e^* has increased (additively) by 5.

   Suggest how to compute a maximum (s,t)-flow f' in G' in $O(n+m)$ time. Briefly explain (i) why your suggested algorithm is correct (ii) why it achieves the desired running time.

   (Hint: You may want to use the (s,t)-flow f as well as the residual graph for G' with respect to f.)

4. (3+2 points)

   (a) Compute the solution to the recurrence relation

   $$a_n = 4a_{n-1} - 4a_{n-2} + 3^n \quad (n \geq 2) \quad \text{with} \quad a_0 = 0 \text{ and } a_1 = 1\,.$$

   (b) Consider a $1 \times n$ strip that we want to fill using two types of tiles:

   - *black* tiles of length 1 (that is, (1×1)-tiles)
   - *white* tiles of length 2 (that is, (1×2)-tiles)

   Let a_n be the number of distinct possible ways to fill the $1 \times n$ strip without two white tiles being placed next to each other. Compute a_1, a_2, a_3 and a_4 and a recurrence relation for a_n for all $n \geq 4$. (You do not need to solve this recurrence relation.)

5. (5 points) Assume that Alice has published modulus $n = 55$, and exponent $e = 7$. Bob sends ciphertext $C = 5$ to Alice. You are eavesdropper Eve and you are interested in Bob's secret message M. Compute Bob's secret message M from ciphertext C. In doing that, write down all of the computational steps that you need to perform in order to obtain Bob's secret message M.

6. For each of the following claims, decide if true or false or if you would rather not give an answer. A correct answer gives $\mathbf{X}$, an incorrect answer $-\mathbf{X}$ and not giving an answer **0 points** (minimum total number of points for Question 6 is 0 points). **Instead of guessing, it may be better not giving an answer.**

   - Students registered under course code 202500366 (3EC standalone ADM course) need to answer all 5 subquestions, and each subquestion is worth $X = 2$ points.
   - **Repeat students** that are registered under course code 202001360 (5EC course, combining ADS and ADM parts), will be graded only on subquestions (a) – (d) and each exercise is worth $X = 2.5$ points.

   (a) Consider a capacitated network $G = (V, E, c)$ where $c : E \to \mathbb{Z}_{\geq 0}$ are the edge capacities. Assume that G has a unique minimum s-t cut. Then G must also have a unique maximum flow.

   True . . . . ☐
   False . . . . ☐
   I prefer to not give an answer . . . . ☒

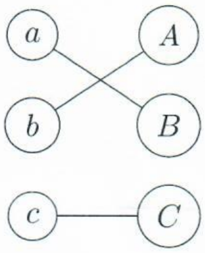
(b) Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$. Suppose we multiply the weight of every edge in the graph by the same constant $c > 1$. Then the shortest path between any pair of vertices remains the same (only its weight will be higher than before).

True . . . . ☒
False . . . . ☐
I prefer to not give an answer . . . . ☐

(c) A connected simple graph is Eulerian *if and only if* none of its edges is a bridge.

True . . . . ☒
False . . . . ☐
I prefer to not give an answer . . . . ☐

(d) Consider the depicted matching M and corresponding preference lists. Then M is a stable matching.



$B >_a A >_a C$
$A >_b B >_b C$
$A >_c B >_c C$
$a >_A b >_A c$
$b >_B a >_B c$
$b >_C c >_C a$

[Reminder: $x >_z y$ indicates that z prefers to be matched with x over y.]

True . . . . ☒
False . . . . ☐
I prefer to not give an answer . . . . ☐

(e) ($*$) Consider a capacitated network $G = (V, E, c)$, where V is the set of vertices, E is the set of directed edges, and $c : E \to \mathbb{Z}_{\geq}0$, are the edge capacities. If some edge $e \in E$ is not part of any minimum cut of G, then there exists a maximum flow f in G such that $f(e) = 0$.

True . . . . ☐
False. . . . . ☐
I prefer to not give an answer . . . . ☒
I am a repeat student (and registered under 202001360) . . . . . ☐