Exam 1 ADS, Module 7, Codes 202001360 & 202001364

Algorithms and Data Structures

Monday March 31, 2025, 8:45 - 9:45

All answers need to be motivated! You are allowed to use a handwritten cheat sheet (A4, both sides) during the exam.

For information: this exam is the ADS part. The entire exam consists of two parts:

Algorithms & Data Structures (ADS)	1h	(30 points) **today's test**
Discrete Mathematics (DM)	2h	(60 points) **last Friday's test**

The total is 30+60=90 points. Your grade is $1 + 0.1x$, x being the number of points, rounded to one digit. That means you need 45 points to get a 5.5.

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1. (10 points)

   (a) Consider this sorting algorithm that sorts an array *arr* of integers:

```
def Sort(arr):
    for i in range(1,len(arr)):
        key=arr[i]
        j=i-1
        while j>=0 and arr[j]>key:
            arr[j+1]=arr[j]
            j=j-1
        arr[j+1]=key
```

   Give the asymptotic time complexity of this algorithm, expressed in the number of comparisons (and motivate your answer). Is this an in–place algorithm?

   (b) Suppose that the number of steps of an algorithm $T(n)$ with an input of n, has the recurrence relation

$$T(n) = 4T(n/2) + n^2$$

   What is the asymptotic complexity class of this algorithm?

2. (10 points)

   (a) Suppose we have a minheap with at least three elements. What is the index of the smallest element bigger than the minimum in this minheap? Explain your answer.

   (b) Given a non-empty binary search tree with unique elements. Give an algorithm (in words or in pseudocode) that yields the smallest element bigger than the minimum.

3. (10 points)

   A number of tracks are to be burnt on a CD. Assume that a choice can be made from n tracks, with a track time of t_i minutes for track i. The CD is to be filled for as far as possible, i.e. with the maximum possible number of minutes of music. Assume that a CD contains at most 80 minutes of music (if it is completely filled).

   (a) Suppose that $C(i,k)$ is the minimum remaining time (i.e. the number of unused minutes) when k minutes are to be filled with tracks from the collection $\{1,\ldots,i\}$. Explain that

   $$C(i,k) = min\{C(i-1,k), C(i-1,k-t_i)\}$$

   where $C(i,k) = \infty$ for $k < 0$.

   What is the value of $C(0,k)$ for $k \geq 0$?

   (b) Give a polynomial algorithm based on dynamic programming that calculates the maximum number of minutes that can be burnt on the CD.