Test Linear Structures 2.
Applied Mathematics, 2024-1B: Structures and Systems

This exam consists of 10 problems which are divided into two parts:
Grasple (digital): 8 problems
Open Questions (written): 2 problems.

Grasple (This morning)
Enter your answers in Grasple in the required form. Follow the instructions precisely.
For the statements, you choose one of three options: true (T), false (F), or no answer (N).
For each correct T or F you will receive (partial) points.
One incorrect T or F results in zero points for that entire question.
Total score for Grasple: 40 points.
Required score: 20 points.

Open Questions (Now)
Write the solutions following two of the four steps.
Step 3. Execute the plan.
Step 4. Analyze your solution and the answer.
Think, for example, of the following questions:
– Can you interpret the solution and/or the result intuitively?
– Does the solution/result make sense in special cases?
– Which role each condition played in the solution?
– Could you relax some assumptions of the problem?
– Is the problem related to other problems or results?
Total score for Open Questions: 40 points.
Step 3: 80%, Step 4: 20%.
Required score: 20 points.
Grade: 1+9(number of points)/80.

A (graphical) calculator is not needed and is **not allowed** at the exam.

PART 2: Open questions.

For each exercise, you can combine parts a) and b) in step 4. That is, you only need to write one reflection (consisting of multiple observations) for exercise 9. You also need only one reflection (consisting of multiple observations) for exercises 10.

9. [20pt] Consider a complex vector space V with dimension n. Consider a linear operator T on V. Recall that the book uses the word 'nullspace' instead of 'kernel'.

(a) Prove that $\ker(T^2 + I)$ is T-invariant.

(b) Prove that if T has n distinct **real** eigenvalues, then $\ker(T^2 + I) = \{0\}$

10. [20pt] Consider a finite-dimensional real inner product space V. Consider a linear operator T on V.

(a) Prove that TT^* is diagonalizable.

(b) Prove that all eigenvalues of TT^* are non-negative.