Hand-out Scheduling 2025

Notation

Many scheduling problems can be described by a three field notation $\alpha|\beta|\gamma$:

- α describes the machine environment
- β describes the job characteristics
- γ describes the objective function to be minimized

Machine environment:

- Single machine ($\alpha = 1$)
- Identical parallel machines ($\alpha = P$ or $\alpha = Pm$)
  - m identical machines (if $\alpha = P$, m is input, else m is a constant)
  - Each job is a single operation which may be processed on any machine for p_j time units.
- Uniform parallel machines ($\alpha = Q$ or $\alpha = Qm$)
  - m parallel machines with speeds s_i
  - $p_{ij} = p_j/s_i$
  - Each job has to be processed by one of the machines.
- Unrelated parallel machines ($\alpha = R$ or $\alpha = Rm$)
  - m different machines in parallel
  - $p_{ij} = p_j/s_{ij}$ with s_{ij} the speed of job j on machine i
  - Each job has to be processed by one of the machines.
- Job shop ($\alpha = J$ or $\alpha = Jm$)
  - Each job has its individual predetermined route to follow.
  - A job does not have to be processed on each machine.
  - A job can visit machines more than once.
- Flow shop ($\alpha = F$ or $\alpha = Fm$)
  - m machines in series
  - All jobs follow the same route: first machine 1, then machine 2, etc.
- Open shop ($\alpha = O$ or $\alpha = Om$)
  - Each job has to be processed on each machine once.
  - Processing times may be 0.
  - No routing restrictions (this is a scheduling decision).

Job characteristics:

- Processing time p_{ij} of operation (i, j) or p_j of job j
- Release date r_j of job j (earliest starting time)
- Due date d_j of job j (committed completion time)
- Weight w_j of job j (importance)
- Preemption: $pmtn$
- Precedence constraints: $prec$

Objective function:

- Makespan ($\gamma = C_{\max} = \max\{C_1, \ldots, C_n\}$ with C_j completion time job j)
- Maximum lateness ($\gamma = L_{\max} = \max\{L_1, \ldots, L_n\}$ with $L_j = C_j - d_j$ lateness job j)
- Total (weighted) completion time ($\gamma = \sum_j (w_j) C_j$)

- Total (weighted) tardiness ($\gamma = \sum_j (w_j) T_j$ with $T_j = \max\{0, L_j\}$ tardiness job j)
- (Weighted) number of tardy jobs ($\gamma = \sum_j (w_j)\, U_j$ with $U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$ unit penalty job j)

Single machines

$1 \| \| \sum C_j$	SPT rule is optimal
$1 \| \| \sum w_j C_j$	WSPT-rule is optimal
$1 \| prec \| f_{max}$	Lawler's algorithm is optimal
$1 \| \| L_{max}$	EDD rule is optimal
$1 \| prec \| L_{max}$	EDD with modified due dates is optimal
$1 \| r_j \| L_{max}$	NP-hard in the strong sense
$1 \| pmtn,\ r_j \| L_{max}$	Preemptive EDD is optimal
$1 \| r_j, d_j < 0 \| L_{max}$	EDD is 2-approximation
$1 \| \| \sum U_j$	Algorithm $1 \| \| \sum U_j$ is optimal
$1 \| \| \sum w_j U_j$	NP-hard
$1 \| \| \sum T_j$	NP-hard
$1 \| \| \sum w_j T_j$	NP-hard in the strong sense

Parallel machines

$P2 \| \| C_{max}$	NP-hard. PTAS exists.
$P \| \| C_{max}$	NP-hard in the strong sense
$P \| \| C_{max}$	List scheduling is $\left(2 - \frac{1}{m}\right)$ - approximation
$P \| \| C_{max}$	LPT rule is $\left(\frac{4}{3} - \frac{1}{3m}\right)$ - approximation
$P \| pmtn \| C_{max}$	Solution $O \| pmtn \| C_{max}$ based on LP-relaxation is optimal
$Q \| pmtn \| C_{max}$	Solution $O \| pmtn \| C_{max}$ based on LP-relaxation is optimal
$R \| pmtn \| C_{max}$	Solution $O \| pmtn \| C_{max}$ based on LP-relaxation is optimal
$P \| r_j \| C_{max}$	List scheduling is $\left(2 - \frac{1}{m}\right)$ – approximation
$P \| \| \sum C_j$	SPT rule is optimal
$Q \| \| \sum C_j$	Modified SPT rule is optimal
$R \| \| \sum C_j$	Polynomially solvable by solving assignment problem
$P2 \| \| \sum w_j C_j$	NP-hard
$P \| \| \sum w_j C_j$	NP-hard in the strong sense
$P \| \| \sum w_j C_j$	WSPT is $\frac{1}{2}(1 + \sqrt{2})$-approximation
$R \| \| \sum w_j C_j$	NP-hard in the strong sense
$R \| \| \sum w_j C_j$	LP rounding is 2-approximation

Shop models

$F2\|\|C_{max}$	Johnson's algorithm is optimal
$F3\|\|C_{max}$	NP-hard in the strong sense
$O2\|\|C_{max}$	Algorithm $O2\|\|C_{max}$ is optimal
$O3\|\|C_{max}$	NP-hard
$O\|pmtn\|C_{max}$	Algorithm $O\|pmtn\|C_{max}$ is optimal
$J2\|\|C_{max}$	Algorithm $J2\|\|C_{max}$ is optimal
$J\|\|C_{max}$	NP-hard in the strong sense; Shifting bottleneck heuristic works well

Scheduling with uncertainty

- Uncertain instance size (e.g. number of jobs)

→ on-line scheduling

- Uncertain data (e.g. processing times)

→ stochastic scheduling

Online $1|r_j|\sum C_j$:

SPT	Non-constant competitive ratio
DSPT	2-competitive
α-scheduler	
• Fixed α	$1 + \frac{1}{\alpha}$ - competitive
• Random $\left(f(\alpha) = \frac{e^{\alpha}}{e-1}\right)$	$\frac{e}{e-1}$ - competitive

- The Shortest Expected Processing Time (SEPT) rule minimizes the expected sum of the completion times in the class of static list policies as well as in the class of dynamic policies.
- The WSEPT (weighted shortest expected processing time $\mathbb{E}[p_j]/w_j$ first) rule minimizes the expected weighted number of tardy jobs in the class of static list policies and dynamic policies when job j has exponentially distributed processing time P_j with rate λ_j and deterministic due date d.
- The EDD rule minimizes the expected maximum lateness for arbitrarily distributed processing times and deterministic due dates in the class of static list policies and dynamic policies.

Operating room scheduling

- Operating room scheduling and number of required beds
- Minimize waiting time emergency surgery; BIM (break-in-moments) problem
- Operating room rescheduling

Railway scheduling

- Line planning
- Timetabling
- Rolling stock planning
- Crew scheduling & rostering

Scheduling in energy management

- General background energy transition
- Smart grids
- Decentralized energy management
- Planning of devices
  - Electric vehicles
  - Batteries