Resit Mastermath course 'Scheduling' 2025 16-6-2025

The exam consists of 5 questions worth 18 points each. Your grade is given by $1 + \frac{p}{10}$, where p is the total number of points obtained.

Note: You are only allowed to use the handout distributed by the lecturers. **Good luck!**

Question 1 *(18 points)*:

Consider the following instance of problem $O3|pmtn|C_{max}$:

$$n = 3, \qquad p = \begin{pmatrix} 4 & 4 & 2 \\ 5 & 1 & 5 \\ 2 & 5 & 4 \end{pmatrix}$$

where each column provides the processing times of a job on the three machines.

Apply the optimal algorithm presented in the lecture to this instance. Explain the different steps, provide the results of these steps, sketch the resulting optimal schedule, and provide the makespan.

Question 2 *(18 points)*:

Give an optimal polynomial time algorithm for the problem $Q|p_j = p|\sum w_j C_j$. Prove both its optimality and that it has polynomial running time. You do not need to prove its exact running time, only that it is polynomial.

Question 3 *(18 points)*:

Consider the following algorithm for $P||C_{\max}$:

1. Order the jobs $1, \dots, n$, such that $p_1 \geq \dots \geq p_n$.
2. For j from 1 to n:
   assign job j to the smallest index machine i such that its load L_i does not exceed $\max\left\{\max_j p_j\,,\ \alpha \sum p_j\right\}$. If no such machine exists, assign j to the least loaded machine.

a) Prove that, for $\alpha = \frac{2}{m}$, the algorithm is a 2-approximation algorithm. *(9 points)*

b) Let $p_n \leq \frac{1}{3} C_{\max}^{\text{OPT}}$ be given, i.e., the smallest processing time is not larger than one third of the optimal makespan. Give a value for α that gives the best possible approximation factor. Prove that the (corresponding) approximation factor holds. You do not need to prove that it is the best possible. *(9 points)*

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Question 4 *(18 points)*:

Prove that any deterministic on-line algorithm for problem $1|r_j|\sum C_j$, in which all job characteristics become known at the release date, has a competitive ratio of at least 2.

Note: jobs with $p_j = 0$ are allowed.

Question 5 *(18 points)*:

Prove that the discrete EV charging problem is NP-hard.

Note: NP-hardness for this problem even holds if all sets Z_n are equal, but the proof is (a lot) easier when the sets Z_n are not all equal.

END OF THE EXAM