Exam Mastermath course 'Scheduling' 2025 **26-5-2025**

The exam consists of 5 questions worth 18 points each. Your grade is given by $1 + \frac{p}{90}$, where p is the total number of points obtained.

Note: You are only allowed to use the handout distributed by the lecturers. **Good luck!**

Question 1 *(18 points)*:

Consider the following instance of the problem $O2||C_{max}$:

$$n = 3, \qquad p = \begin{pmatrix} a & 1 & 4 \\ b & 2 & 1 \end{pmatrix}$$

with a and b non-negative integers.

Apply the optimal algorithm presented in the lecture to this instance. Explain the different steps, provide the results of these steps and give the optimal schedules (in terms of order of jobs per machine and order of machines per job) dependent on a and b. Finally, express the optimal makespan as a function of a and b.

Question 2 *(18 points)*:

Consider the problem $1||\sum \frac{1}{-C_j}$ (objective is the sum of $\frac{1}{-C_j}$ over all jobs j), where $p_j > 0$ for all jobs j.

Give a scheduling rule (algorithm) for this problem and prove that it leads to an optimal solution.

Question 3 *(18 points)*:

a) Consider the following instance of $P2||C_{\max}$ with jobs with stochastic processing times. There are eight jobs ($n = 8$). Seven have a deterministic processing time, while one has a stochastic processing time:

$$p_j = 3 \text{ for } j \in \{1,2,3,4\}, \qquad p_j = 4, \text{ for } j \in \{5,6,7\}$$

$$p_8 = \begin{cases} 0, & \text{with probability } \frac{1}{2} \\ 12, & \text{with probability } \frac{1}{2} \end{cases}.$$

Give a static list-scheduling policy that is optimal among dynamic list-scheduling policies for this instance, and prove that it is. *(9 points)*

b) For the $Q2||C_{\max}$ problem with jobs with stochastic processing times, prove that an optimal non-anticipatory policy, in general, does not produce an optimal schedule for each realization of the processing times. *(9 points)*

HINT: There exist sufficient examples with just one job with stochastic processing time with two values in the support of its processing-time distribution.

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Question 4 *(18 points)*:

Prove that list scheduling is an α- approximation for the problem $P|r_j|C_{\max}$.

There are three options for α with increasing difficulty for increasing number of points. Choose one:

(8 points): $\alpha = \left(3 - \frac{1}{m}\right)$

(13 points): $\alpha = 2$

(18 points): $\alpha = \left(2 - \frac{1}{m}\right)$

Question 5 *(18 points)*:

Prove that the BIM problem, in which we determine the order of elective surgeries in their preassigned operating room to minimize the maximum waiting time for emergency surgeries, is strongly NP-hard.

END OF THE EXAM