Exam Stochastic Models – Queueing
Wednesday June 25, 2025, 13.45 - 15.45 hours.

Books, notes, etc. are **not** allowed. An ordinary (scientific) calculator is allowed, but a programmable or graphic calculator ('GR') is **not** allowed.

This exam consists of **two exercises.**
Put your name and student number on **all** your work.
Also hand in the question paper.
Total points: 90. Grade = 1 + obtained points / 10
Motivate all answers.

1. (44 points) Customers arrive at a service center according to a Poisson process with rate λ per hour. The service center has 2 queues, queue 1 and queue 2. The service center has 1 server available that may either serve a customer from queue 1 or from queue 2. If the server is serving a customer at queue i, the service time is exponentially distributed with service rate $\mu_i = \mu$ per hour, $i = 1, 2$. If the queues are not of equal length, then an arriving customer will always choose to join the shortest queue; if both queues are of equal length, then an arriving customer will always join queue 1. If the queues are not of equal length, then the server will always serve a customer from the longest queue; if the queues are of equal length then the server will serve a customer from queue 1; to achieve this policy, if needed the server will switch from serving a customer at queue 2 to serving a customer at queue 1. When the server later returns to the 'abandoned' customer at queue 2, its service will start afresh. Let $X(t)$ be the stochastic process describing the number of customers in both queues at time t; we denote its states by (n_1, n_2), $n_1, n_2 = 0, 1, 2, \ldots$. Let $P(n_1, n_2)$ denote the steady state distribution of $X(t)$.

(a)–5 pt Why is $\{X(t),\ t > 0\}$ a Markov chain?

(b)–5 pt Give the stability condition for $\{X(t),\ t > 0\}$. Motivate your answer.

(c)–6 pt Give the transition diagram of $\{X(t),\ t > 0\}$, including a motivation for the transition rates in this diagram.

(d)–6 pt Give the equations to determine this steady state distribution $P(n_1, n_2)$.

(e)–6 pt Show that $P(1,1) = \left(\frac{\lambda}{\mu}\right)^2 P(0,0)$.

(f)–6 pt Express $P(n,n)$ in λ, μ, and $P(0,0)$, $n = 1, 2, 3, \ldots$.

The following may be answered in terms of the $P(n_1, n_2)$ and the parameters λ, and μ.

(g)–5 pt What is the utilisation of the server? Motivate your answer.

(h)–5 pt Give an expression for the average number of customers in queue 2 (waiting plus service). Motivate your answer.

P.T.O.

2. **(46 points)** Consider an open network of two stations. Every station contains a single server, and all customers arriving at a station may enter the station. Service is in order of arrival. The service times at stations 1 and 2 have exponential distributions with rates μ_1, and μ_2, respectively. The external arrival rate of customers to station 1 and station 2 is γ_1 and γ_2, respectively. Customers that complete service at station 1 route to station 2. A fraction $1-p$ of customers served at station 2 routes to station 1 and a fraction p leaves the network.

(a)–5 pt Give the traffic equations and solve the traffic equations.

(b)–4 pt Give the stability condition for the network. Motivate your answer.

(c)–4 pt Give the distribution of the number of customers at each station. Motivate your answer.

(d)–4 pt Give the joint distribution of the number of customers at the two stations. Motivate your answer.

(e)–5 pt For each station, give an expression for the mean number of customers at the station and an expression for the mean sojourn time at the station.

(f)–5 pt Give an expression for the mean sojourn time of a customer in the network. Motivate your answer.

(g)–5 pt Give the probability that an arriving customer finds the system empty. Motivate your answer.

Now assume that both stations are infinite server stations. The service times at stations 1 and 2 have exponential distributions with rates μ_1, and μ_2, respectively. Assume that $\gamma_2 = 0$, and $p = 1$.

(h)–5 pt Give the distribution of the number of customers at each station. Motivate your answer.

(i)–4 pt Give the distribution of the total number of customers in the network. Motivate your answer.

(j)–5 pt Give the distribution of the sojourn time of a customer in the network. Motivate your answer.