Exam Game Theory (191521800)

University of Twente

November 07, 2024, 8:45-11:45h

This exam has 7 exercises.
Motivate all your answers! **You may not use any electronic device.**
You are allowed to bring your own cheat sheet (1 A4, double-sided).

Noncooperative Game Theory

1. (2+2 points) Consider the (symmetric) bimatrix game given by

$$(A, B) = \begin{pmatrix} -5, -5 & 10, 3 \\ 3, 10 & 8, 8 \end{pmatrix}$$

   (a) Compute all Nash equilibria of this game.

   (b) Write down all conditions that define the correlated equilibria of this game, and exhibit a correlated equilibrium that is not a Nash equilibrium.

2. (1+2+1+1 points)

   Three items I_1, I_2, and I_3 have different worth for two players 1 and 2, given by the following table:

	I_1	I_2	I_3
Worth for player 1	1	2	3
Worth for player 2	2	3	1

   Player 1 starts with choosing an item. After that, player 2 chooses one of the remaining items. Finally, player 1 gets the item that is left.

   (a) Draw the decision tree of this extensive form game.

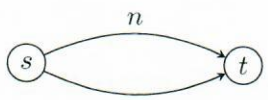
   (b) Give the strategic form representation of this extensive form game.

   (c) Determine a subgame perfect equilibrium.

   (d) Give Nash equilibrium that is not a subgame perfect equilibrium.

3. (3 points)

   Consider the simple road network shown below.



   There are n cars going from s to t. The upper road takes n minutes (independent of the number of cars that use this road), the lower road takes $2x$ minutes if x cars take it.

   What is the price of anarchy for this instance? (You may assume that $4|n$).

Cooperative Game Theory

4. (2+3+1 points) Consider the following three player cooperative game (N, v).

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	2	5	4	15	18	14	24

   (a) Is the game a convex game? Is it super-additive? Give proofs.

   (b) Compute and depict the core $C(N, v)$, as well as the domination core $DC(N, v)$ of that game. What are the extreme points (vertices) of the core?

   (c) What is the maximal value $v(\{2,3\})$ such that $C(N, v) \neq \emptyset$?

5. (4+2 points) Consider the following solution value ψ defined for a cooperative game (N, v) with $|N| = n$. For all $i \in N$, let i's payoff be the average of all marginal contributions of i,

$$\psi_i(N, v) := \frac{1}{2^{n-1}} \sum_{S: i \notin S} \left(v(S \cup \{i\}) - v(S)\right).$$

   (a) Show that ψ is symmetric, additive and fulfils the null player property, but may fail to be efficient (in general).

   (b) Show that ψ equals the Shapley value when $n = 2$.

Stochastic Game Theory

6. (4+3 points) Below you see the graphical representation of a zero-sum stochastic game with an infinite horizon, the discounted-reward criterion and $\beta = 4/5$.

5 $(0,1)$	-4 $(\frac{1}{2}, \frac{1}{2})$
1 $(0,1)$	2 $(\frac{1}{2}, \frac{1}{2})$

state 1

0   0, 1)

state 2

   (a) Determine the value vector of this game and the optimal strategies of the players. (*Hint:* the matrix game $R_\beta(1, \mathbf{v}_\beta)$ has no saddle points.)

   (b) Next, consider the same game with a *finite* horizon. Which type of strategies are optimal for the players, and how does this type differ from the type in part (a)?

7. (2+3 points)

   (a) Mention one similarity and one difference between noncooperative games and stochastic games.

   (b) Consider an irreducible stochastic game with the average reward criterion. Let $v\mathbf{1}_N = v_\alpha(\mathbf{f}, \mathbf{g})$ and $\mathbf{w} = (I - P(\mathbf{f}, \mathbf{g}) + Q(\mathbf{f}, \mathbf{g}))^{-1}(\mathbf{r}(\mathbf{f}, \mathbf{g}) - v\mathbf{1}_N)$. Show that the pair $(v, \mathbf{w})$ satisfies

$$\mathbf{w} + v\mathbf{1}_N = \mathbf{r}(\mathbf{f}, \mathbf{g}) + P(\mathbf{f}, \mathbf{g})\mathbf{w}.$$

Total: 36 points. Grade = (points + 4) / 4