Exam Continuous Optimization

24 January 2022, 13.30-16.30

This closed-book exam consists of 5 questions. Please start each question on a new page, write legibly, and hand in your work with the solutions in the correct order. Good luck!

$$\sqrt{1.}$$
 (5 points) Let

$$f(x_1, x_2) = x_1^2 - x_2^2.$$

Compute the Newton direction $\Delta x_{\rm nt}$ of f at (1,2) and show this is not a descent direction.



 $2.\ (10\ points)$ Consider the equality constrained least squares problem

minimize
$$||Ax - b||_2^2$$
 subject to $Cx = d$.

Explain how (and why) we can use the KKT optimality conditions to solve this problem by solving a single linear system.

Check?

3. (10 points) Derive the Lagrangian, Lagrange dual function, and the Lagrange dual problem of the following optimization problem in x and y:

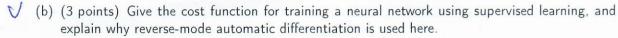
$$\mathsf{minimize} \quad -\sum_{i=1}^m \log(y_i)$$

subject to
$$y = b - Ax$$

√4. (a) (7 points) Consider the function

$$F(x,y) = x^3 + \frac{1}{xy},$$

where we view addition, multiplication, taking the reciprocal, and taking the third power as elementary functions. Show how $\nabla F(1,2)$ is computed using reverse-mode automatic differentiation by drawing the appropriate diagrams.



5. Consider the barrier method for an optimization problem of the form

$$\label{eq:f0} \begin{aligned} & \text{minimize} & & f_0(x) \\ & \text{subject to} & & f_i(x) \leq 0, \quad i = 1, \dots, m, \end{aligned}$$

where the functions f_0, \ldots, f_m are convex and twice continuously differentiable. Assume the problem has an optimal solution x^* with objective p^* . Assume furthermore the problem is strictly feasible.

 \sim (a) (7 points) Show that if $x^*(t)$ is optimal for the centering problem with parameter t, then

$$f_0(x^*(t)) - p^* \le \frac{m}{t}.$$

 \bigvee (b) (3 points) Suppose m=1000 and we apply the barrier method with parameters $\mu=2, \, \epsilon=1,$ and with 1 as the initial value for t. After approximately how many outer iterations does the barrier method terminate?