Complex Function Theory - Exam
Course code 201500405

Date	:	Monday 8 June 2026
Place	:	SC 1
Time	:	8:45 – 10:45

All answers must be motivated.
The use of books, notes, calculators or any other electronic device is not allowed.

1. Show that if f is analytic in a domain D and $\mathrm{Im}(f(z))$ is constant in D, then $f(z)$ must be constant in D.

2. Prove that $|e^{i\pi}| = |i^{\pi e}|$.

3. Prove or give a counterexample to the following statement:
   Let Γ be a closed contour. If f is differentiable at every $z \in \Gamma$, then $\int_\Gamma f(z)\,dz = 0$.

4. Let f and g be entire functions that satisfy $|f(z)| < |g(z)|$ for all $z \in \mathbb{C}$. Show that there exists a constant $\lambda \in \mathbb{C}$ such that $f(z) = \lambda g(z)$ for all $z \in \mathbb{C}$.

5. Find the Laurent series (in powers of $z-1$) for the function $\dfrac{1}{z^2 - 3z + 2}$ in the region $0 < |z-1| < 1$.

6. Calculate
$$\text{p.v.} \int_{-\infty}^{\infty} \frac{x}{(x^2+2x+2)(x^2+4)}\, dx.$$

7. Prove that $z^7 - 5z + 3 = 0$ has precisely one root in the disk $|z| < 1$.

Points per exercise:

Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5	Ex. 6	Ex. 7
3	4	3	3	5	5	4

Grade is calculated as $\dfrac{\text{sum of points } + 3}{3}$.