

Complex Function Theory - Resit

Course code 201500405

Date : Monday 29 June 2026
Place : CR 2G
Time : 8:45 – 10:45

All answers must be motivated.

The use of books, notes, calculators or any other electronic device is not allowed.

1. Determine all entire functions $f : \mathbb{C} \rightarrow \mathbb{C}$ that are of the form $f(x + iy) = u(x) + iv(y)$, where u and v are real functions of one real variable.
2. Show that the set of values of $\log(i^2)$ differs from the set of values of $2\log(i)$.

3. Calculate the following contour integral:

$$\int_{\gamma} \bar{z} \, dz,$$

where γ is the straight line segment from $z_1 = 0$ to $z_2 = 1 + i$.

4. Find the Laurent series for the function $f(z) = \frac{1}{z(z-2)}$ in the following regions:

- (a) $D_1 = \{z \in \mathbb{C} : 0 < |z| < 2\}$
- (b) $D_2 = \{z \in \mathbb{C} : |z| > 2\}$

5. Classify the isolated singularities of the function $f(z) = \frac{\sin(z)}{z^2(z^2 + 1)}$. If a singularity is a pole, also determine its order. Make sure to justify your answers.

6. Calculate

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 4)^2 (x^2 + 9)} \, dx.$$

7. Prove that $z + 3 + 2e^z = 0$ has precisely one root in the left-half plane $H = \{z \in \mathbb{C} : \operatorname{Re}(z) < 0\}$.

Points per exercise:

Ex. 1	Ex. 2	Ex. 3	Ex. 4	Ex. 5	Ex. 6	Ex. 7
3	3	3	5	3	5	5

Grade is calculated as $\frac{\text{sum of points} + 3}{3}$.