Exam Complex Function Theory
Code 2024-201500405

Date : Wednesday, June 4, 2025
Place : TL-3130
Time : 08.45 – 10.45

All answers must be motivated.
The use of the book, or lecture notes, summaries, etc. is not allowed.
The use of a pocket calculator or any other electronic equipment is not allowed.

1. Does there exists an analytic function on the domain $D = \{s \in \mathbb{C} \mid \mathrm{Re}(s) > 0\}$ whose real part evaluated at $z = x + iy \in D$ equals x^2, i.e., $u(x, y) = x^2$?

2. (a) Determine the outcome of the following integral

$$\int_\gamma \left[z^3 + \frac{1}{z-4} \right] dz, \tag{1}$$

where γ is an arc of the circle with radius 2 and centre $z = 1$. The arc is running from 3 to -1, see also Figure 1.

(b) Is the outcome of the integral (1) real-valued?

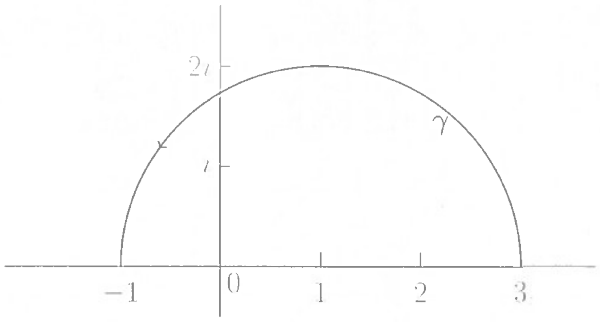

Figure 1: The curve γ.

3. Determine all solutions of the equation $\sin(z) = \cos(z)$.

4. Let f be an entire function and let C_R be the circle $C_R = \{s \in \mathbb{C} \mid |s| = R\}$. Assume that $f(z) \neq 0$ for all $z \in \mathbb{C}$ satisfying $|z| \leq R$.

(a) Prove that

$$\min_{z \in \mathbb{C}, |z| \leq R} |f(z)| = \min_{s \in C_R} |f(s)|. \tag{2}$$

(b) Does the equality (2) still hold when f is allowed to have a zero, z_0, with $|z_0| < R$?

P.T.O.

5. Determine the integral of $\dfrac{e^z}{z^2+4}$ along the circle $C_5 = \{z \in \mathbb{C} \mid |z| = 5\}$, where C_5 is traversed once in the positive direction.

6. Consider the function

$$h(z) = z^2 \cos(1 - z^{-1}).$$

   (a) Determine the singularities of h, and characterize them.

   (b) Determine the zeros of h with their order.

7. Determine the following integral

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{x^2 - 2x + 1}{(x^2+1)(x^2+9)} dx.$$

8. Consider the equation

$$z = 3 - e^{-z}. \tag{3}$$

   (a) Show that the equation (3) possesses exactly one solution in the right half-plane.

   (b) Show that the solution of (3) in the right half-plane is real.

Points[1]

Ex. 1	Ex. 2		Ex. 3	Ex. 4		Ex. 5	Ex. 6		Ex. 7	Ex. 8	
3	a	3	4	a	3	4	a	3	5	a	4
	b	1		b	2		b	3		b	1

[1] Total: $36 + 4 = 40$ points