Systems Theory (Module 2 — code 202200238)

Date:	24-01-2025
Time:	13:45–16:45 (till 12:30 for students with special rights)
Place:	TL-3138
Course coordinator:	Gjerrit Meinsma
Allowed aids during test:	None

1. Consider the differential equation

$$y^{(4)}(t) + 2y^{(3)}(t) + 5y^{(2)}(t) = u(t).$$

   (a) Determine all real homogeneous solutions $y(t)$.

   (b) Now let $u(t) = 10t$. Determine all real solutions $y(t)$ of the above DE.

2. What is the definition of *time constant* of an asymptotically stable DE $y^{(1)}(t) + p_0 y(t) = 0$?

3. For which $c \in \mathbb{R}$ is $\lambda^4 + 2\lambda^3 + 5\lambda^2 + 2\lambda + c$ asymptotically stable?

4. Let $t \in \mathbb{R}$ and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

   (a) Is the system $\dot{x}(t) = Ax(t)$ asymptotically stable?

   (b) Determine e^{At}.

5. Determine a state representation $\dot{x}(t) = Ax(t)+Bu(t), y(t) = Cx(t)+Du(t)$ of the differential equation

$$y^{(2)}(t) - 3y^{(1)}(t) + y(t) = u^{(2)}(t) - 4u^{(1)}(t).$$

6. Let $\alpha \in \mathbb{R}$. Consider

$$\dot{x}(t) = \begin{bmatrix} -\alpha & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t).$$

(a) For which α's is the system controllable?

(b) Determine the reachable subspace.

(c) For which α's is the system observable?

(d) For which α's is the system detectable?

(e) Take $\alpha = 1$. Determine the state feedback $u = -Fx$ that places the two eigenvalues of $A - BF$ at -1 and -3.

7. Three questions.

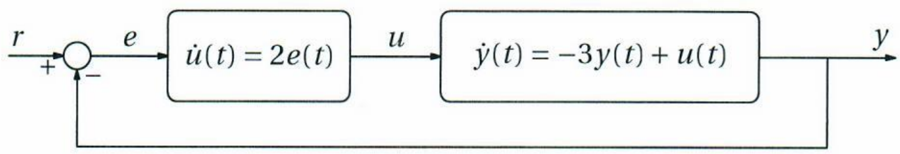
(a) What is the definition of an *observable* system?

(b) The method of *variations of constants* can also be used for time-varying DE's such as

$$\dot{y}(t) + t y(t) = t.$$

Write $y(t)$ as $y(t) = c(t)\mathrm{e}^{-t^2/2}$ and use it to determine all differentiable solutions of this DE.

(c) Consider the following closed-loop system



Here, $r(t)$ is some given function, and $e(t) := r(t) - y(t)$. Suppose that $r(t)$ is constant: $r(t) = r_0$. Show that $\lim_{t\to\infty} y(t) = r_0$.

problem:	1	2	3	4	5	6	7
points:	2+4	1	2	1+6	2	2+2+2+3+2	2+3+2

Exam grade: $1 + 9p/36$.